VI Semester B.A./B.Sc. Examination, May 2017
(NS) (2013-14 and Onwards) (Semester Scheme) (Repeaters)
(Prior to 2016-17)
MATHEMATICS – VII

Time : 3 Hours | Max. Marks : 100

***Instruction**: Answer **all** questions.*

I. Answer **any fifteen** questions. (15×2=30)

1) Find the locus of the point z, satisfying $|z - 1| \geq 2$.

2) Define analytic function. Give an example.

3) Verify whether $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is a harmonic function.

4) Find the orthogonal trajectories of the family of curves $x^2 - y^2 = c$.

5) Define bilinear transformation.

6) Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the line y = x.

7) State Cauchy's integral formula.

8) Evaluate $\int_C \frac{dz}{z(z-2)}$, where C is the circle $|z| = 3$.

9) Evaluate $\int_C \left[(3x+2y)dx + (y+2z)dy - x^2dz\right]$, where C is the curve defined by $x = t$, $y = 2t^2$, $z = 3$, t varying from 0 to 1.

10) Evaluate $\int_0^1\int_0^1 \left(x^2 + y^2\right) dydx$.

11) Show that $\iint_R ydxdy = \frac{4}{5}$, where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.

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12) Find the area of the circle $x^2 + y^2 = a^2$ by double integration.

13) Evaluate $\int_0^1 \int_0^2 \int_1^2 xyz^2 dxdydz$.

14) Show that area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab, using Green's theorem.

15) If V is the volume of a region bounded by a closed surface S, show that

$$\iint_S \vec{r} \cdot \hat{n} \, dS = 3V.$$

16) Evaluate $\iint_S [(x+z)i + (y+z)j + (x+y)k] \cdot \hat{n} \, dS$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 4$ by using Gauss divergence theorem.

17) Give an example of a set which doesn't have any point of accumulation.

18) If $\lim_{n \to \infty} a_n = b$ and $\lim_{n \to \infty} a_n = c$, then prove that $b = c$.

19) Define closure of a set.

20) Define subbase for a topology.

II. Answer **any four** questions. **(4×5=20)**

1) If $\frac{z-i}{z-1}$ is purely imaginary then show that its locus is a circle.

2) If $f(z) = u + iv$ be an analytic function in the domain D of a complex plane then $u = c_1$ and $v = c_2$, where c_1 and c_2 are constants represents orthogonal family of curves.

3) Find the analytic function whose real part is $\frac{\sin 2x}{\cos 2x + \cosh 2y}$.

4) If $f(z) = u + iv$ is analytic, then show that $\left[\frac{\partial}{\partial x} |f(z)|\right]^2 + \left[\frac{\partial}{\partial y} |f(z)|\right]^2 = |f'(z)|^2$.

5) Discuss the transformation $w = e^z$.

6) Find the bilinear transformation which maps $z = \infty, i, 0$ onto $w = 0, i, \infty$ respectively.

III. Answer **any two** questions : (2×5=10)

1) Evaluate $\int_0^{1+i} \left(x^2 - iy\right) dz$ along the curves $y = x$ and $y = x^2$.

2) Show that $\oint_C \frac{e^{2z}}{(z-2)^3} dz = 4\pi i e^4$ where C is the circle $|z| = 3$.

3) State and prove the fundamental theorem of algebra in complex variables.

IV. Answer **any four** questions. (4×5=20)

1) Evaluate $\int_C (x+y+z) ds$ where C is the line joining the points (1, 2, 3) and (4, 5, 6).

2) Change the order of integration and evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dy\, dx$.

3) Show that $\int_0^{2a} \int_0^{\sqrt{2a-x^2}} x^2 dy dx = \frac{5\pi a^4}{8}$ by changing to polar coordinates.

4) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} dxdydz$.

5) Find the surface area of the sphere $x^2 + y^2 + z^2 = a^2$ by using double integration.

6) Evaluate $\iiint_R xyz\, dxdydz$, where R is the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ by transforming into cylindrical polar coordinates.

V. Answer **any two** questions. (2×5=10)

1) State and prove Green's theorem in the plane.

2) Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ taken over the rectangular parallelepiped $x = 0$, $y = 0$, $z = 0$ and $x = a$, $y = b$, $z = c$.

3) Evaluate by Stoke's theorem for the function $\vec{F} = y^2 i + xyj - xzk$, where S is the hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$.

VI. Answer **any two** questions. (2×5=10

1) Prove that the union of any number of open subsets of R^2 is open.

2) Let $X = \{x, y, z\}$ and $\tau = \{X, \Phi, \{x\}, \{z\}, \{x, z\}\}$ then show that τ is a topology on X.

3) Let A and B be any two subsets of the topological space X, then prove that

   i) If $A \subset B \Rightarrow \overline{A} \subset \overline{B}$ and

   ii) $\left(\overline{A \cup B}\right) = \overline{A} \cup \overline{B}$

4) Let (X, τ) be the topological space and A, B any two subsets of X, then prove that

   i) $\text{int}(\phi) = \Phi$

   ii) $\text{int}(X) = X$

   iii) $A \subset B \Rightarrow \text{int}(A) \subset \text{int}(B)$.
